COMPRESSION OF A PLASMA LOOP BY A MAGNETIC FIELD

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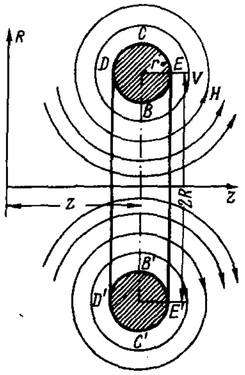
In [1, 2] the conditions for the production and the equilibrium of a plasma loop with a current in an axially-symmetric variable magnetic field were investigated. The plasma ring with its current arose as the result of an electrode less discharge in a gas chamber at pressures from 0.01 to 0.5 mm Hg. In this case only electromagnetic phenomena were taken into account in the theoretical calculation carried out in [1], and gas-dynamic effects were not considered. However, if we suppose that the plasma ring with its current is also a vortex loop, these effects may be allowed for to some extent. Such an assumption is quite natural since vortical motion cannot arise in the zone where the current flows.



We shall first consider the question qualitatively. We shall suppose that a vortex ring with a current (figure) has formed in the gas, and that the magnetic lines of force and the stream lines for the vortex flow are directed as shown in the figure. The current flowing in the ring excites a magnetic field in the ring which is determined by the Biot-Savart law

$$\mathbf{H} = Ic^{-1}\,\mathrm{grad}\,\omega.$$

Here H is the magnetic field strength, c is the velocity of light, ω is the solid angle which the ring subtends at the point where the field strength is measured, I is the current. Since the gas velocity outside the ring, like the magnetic field, is determined by the Biot-Savart law, the velocity and magnetic field strength will be collinear vectors,

$$\mathbf{H} = \varkappa\sqrt{4\pi\rho}\,\mathbf{v}.$$

It may easily be seen that the field strength and velocity at points close to the points B and B' will be greater than at points close to the points C and C', since $\omega(B) > \omega(C)$. The hydrodynamic pressure of the surrounding fluid is calculated by Bernoulli's law,

$$p = p_\infty - {}^1/_2\rho v^2.$$

Thus at points C and C' the hydrodynamic pressure will be greater than at the points B and B'. Correspondingly, the magnetic pressure $H^2/8\pi$ will be greater at the points B and B'. Hence it follows that if the magnetic energy M exceeds the kinetic energy K, the ring will expand, otherwise it will contract. We shall assume that $M > K$. The ring will begin to expand. The velocity at points D and D' will now be less than at points E and E', and consequently the pressure will be greater. As a result of this the ring will begin to move to the right with increasing velocity until the oncoming flow compensates the pressure difference at the points B and B' and C, C', after which the ring will move with constant velocity. If the ring is situated in an external magnetic field, which is in the negative z direction, then this field will create a pressure compressing the ring. In this case if the excess of hydrodynamic pressure and magnetic pressure from the external field compensates the excess pressure due to the self-field, the ring will be in equilibrium. Otherwise it will necessarily move in the direction of its axis. This qualitative analysis shows that it is impossible to compress the ring by an external field in the direction of the z axis, while maintaining its position in space unchanged.

In [3] equations of motion were obtained for thin magnetovortical configurations, which may be applied in the case under consideration. The plasma loop will be characterized by two generalized coordinates R and z (figure).

The equations of motion of the loop may be written in the form

$$-2\pi R\rho\Gamma\frac{dz}{dt} = -\frac{\partial K}{\partial R} + Q_1, \qquad 2\pi R\rho\Gamma\frac{dR}{dt} = Q_2. \quad (1)$$

Here ρ is the density, K is the kinetic energy of cyclic motion, Γ is the value of the circulation, Q_1 and Q_2 are generalized forces corresponding to the coordinates R and z.

Remark. Equations (1) were obtained on condition that the rings are very thin, and so, in view of their small mass, the kinetic energy of the rings themselves may be neglected [3] in comparison with the kinetic energy of cyclic motion of the surrounding fluid. In the case being considered, the kinetic energy of cyclic motion and magnetic energy of the ring current are, respectively,

$$K = \frac{1}{2}\rho\Gamma^2 R\left(\ln\frac{8R}{r} - 2\right), \qquad M = \frac{2\pi R}{c^2}I^2\left(\ln\frac{8R}{r} - 2\right).$$

Here r is the radius of cross section of the loop. The generalized force Q_1 has two components

$$Q_1' = \frac{\partial M}{\partial R}, \qquad Q_1'' = \frac{2\pi R}{c}IH_z.$$

The first of these is caused by the self-current, the second by the external field. H_z is the axial component of the external magnetic field strength at the orbit of the loop. The magnetic energy of the self-current M does not depend on z, and so

$$Q_2 = -\frac{2\pi R}{c}IH_R.$$

Here H_R is the radial component of the external magnetic field at the orbit of the loop.

The current strength in the loop depends on the current in the external windings as follows:

$$LI = -L_{12}I_1 = -\pi R^2 c\langle H\rangle, \qquad \langle H\rangle = \frac{2}{R^2}\int_0^R H_z x\,dx.$$

Here L is the self-induction coefficient of the loop, L_{12} is the coefficient of mutual induction of the winding and the loop, I_1 is the current strength in the external windings, $\langle H\rangle$ is the mean value of the field strength H_z inside the loop.

Summarizing the results which have been obtained, we write system (1) in the following form:

$$2\pi R\rho\Gamma\frac{dz}{dt} = \frac{1}{2}\rho\Gamma^2\left(\ln\frac{8R}{r} - 1\right) - $$
$$- \frac{2\pi^3 R^4}{L^2}\langle H\rangle^2\left(\ln\frac{8R}{r} - 1\right) + \frac{2\pi^2 R^3}{L}\langle H\rangle H_z,$$
$$2\pi R\rho\Gamma\frac{dR}{dt} = -\frac{2\pi^2 R^3}{L}\langle H\rangle H_R. \quad (2)$$

The loop will be in a state of equilibrium if

$$H_R = 0,$$

$$H_z = \frac{\pi R^2}{L} \langle H^2 \rangle \left(\ln \frac{8R}{r} - 1 \right) - \frac{\rho \Gamma^2 L}{4\pi^2 R^3 \langle H \rangle} \ln \left(\frac{8R}{r} - 1 \right). \quad (3)$$

In order to compress the loop into a ball while maintaining its position in space, an axial magnetic field must be created which satisfies the second of conditions [3], and a radial field which satsifies the first condition of [3]. In the case when the density of the surrounding gas is low $\rho \ll 1$ condition (3) is similar to the condition obtained in [1].

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